1* Let $\emptyset \neq A \subseteq \mathbb{R}$, bounded with $x := \sup A \in \mathbb{R}$. Show that $\exists$ a seq (x_n) in A such that $\lim_n x_n = x$. Moreover, if $x \notin A$ show that you can have your (x_n) satisfying additionally that $x_n < x_{n+1}$ $\forall n$.

2* Let (a_n) be a bounded sequence, and

$$t_n = \inf\{a_m : m \geq n\} = \inf\{a_n, a_{n+1}, a_{n+2}, \cdots\}$$

$$s_n = \sup\{a_m : m \geq n\} = \sup\{a_n, a_{n+1}, a_{n+2}, \cdots\}.$$

Show that $(t_n), (s_n)$ are monotone and

$$\lim_n t_n = \sup\{t_n : n \in \mathbb{N}\} \leq \inf\{s_K : K \in \mathbb{N}\} = \lim_K s_K.$$

3* Let $(a_n), (t_n), (s_n)$ be as in Q9. Show that (a_n) converges iff $\lim_n t_n = \lim_n s_n$.

$\lim_n t_n$ is usually denoted by $\liminf_n a_n$ (lower limit of (a_n))

$\lim s_n$ — — ——————— $\limsup_n a_n$ (upper limit of (a_n))